

# Group Work: Cardinality

CIS 300 Foundations of Computer Science

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## Introduction

This assignment is about using *propositional* logic, truth tables, and definitions.

## Assignment Goals

Learning Outcomes

- Prove some sets are countable.

## Procedure

**Assign Roles** (If possible, take a role that you have not taken before or one that you have not held recently.)

**Manager**

**Speaker**

**Reflector**

**Recorder** (combine with **Reflector** if there are fewer than four members.)

1. Consider an *alphabet*  $\{\spadesuit, \heartsuit, \diamondsuit, \clubsuit\}$ . List five (5) **strings** across this alphabet, each with a different length.
2. What is the *definition* of **countable**?
3. Describe how you would prove that an *infinite set* is **countable**.
4. Justify that each of the following is **countable**.
  - (a)  $\emptyset$
  - (b)  $\{x \mid x = 3^z \text{ for } z \in \mathbb{Z}^+\}$
  - (c)  $\{\omega \mid \omega \text{ is a valid English word}\}$
  - (d)  $\mathbb{Z}^{\geq 0}$

- (e)  $\{a, b, c, d, e, \dots, x, y, z\}$
- (f)  $\mathbb{Z}^{>0}$

5. **Prove** that  $\mathbb{Z}^{\geq 0}$  is **countable**.

6. Write the first 16 elements of  $\{0, 1\}^*$  in the order we use when proving the set is **countable**.

7. Let  $E ::= \{\text{English Alphabet}\}$ , all in lower-case.

(a) What is  $|E^0|$ ?

(b) List the elements of  $E^1$ .

(c) What is  $|E^4|$ ?

(d) Describe  $E^5$  in an English sentence.

(e) Describe, in English,  $\bigcup_{i=0}^5 E^i$ .

(f) Is  $\bigcup_{i=0}^5 E^i$  a **language**?

(g) Describe, in English,  $E^*$ .

(h) Is  $E^*$  **finite**?

(i) Give your best guess: Is  $E^*$  **countable**? Justify your answer.