

# gChryssippus - Deduction Through the Ages

2024-01-30

## Introduction

This group assignment continues our class work with the **Deduction Through the Ages** project. You will work, in your group, on the *Five Indemonstrable Forms*.

[Based in part on documents found on [cspogil.org](http://cspogil.org).]

## Note

- By **design**, this assignment includes questions about topics that you have not yet seen in lecture. This encourages *creative* exploration and *constructing* your own mental models.
- Your group is expected to work together on *each* question: the goal is **not** to fill in the “right” answers as quickly as possible but rather to take the time to read, think, and discuss the material **together**.
- Constructing your own model is **short-circuited** by reading or parroting another thinker’s answer. This means you should, as much as possible, use *recall* of class material, looking up definitions in *notes* or the *textbook* when necessary, and *avoid* just typing the question into a search engine on the Web. If you want more information, feel free to search **after** the group work is finished.
- Each student in the group has more or less experience with the topics in the assignment. If you have greater familiarity with the topics, *please* hold back a little bit so others can engage with building their own knowledge. You, too, must engage because there is always more to learn.

## Assignment Goals

**Learning Outcomes** After completing this group assignment, each student is expected to be able to

- Differentiate between the *inclusive* and *exclusive* versions of **or**.
- Describe all *five* argument forms attributed to Chryssippus.
- Have a hypothesis about rewriting **implications** in terms of **and**, **or**, and **xor** (exclusive or).

## Procedure

**Get out paper.** The group will turn in *one* document. Make sure all participating members’ names are on the page.

**Copy each question before the answer.** This documents the answer and makes the page a stand-alone study guide.

**Assign Roles.** Students should take roles they have not held recently (or, perhaps, ever):

**Manager** Move discussion forward.

**Recorder** Writes the report that will be turned in.

**Reflector** Monitor that everyone gets heard and is caught up. (This is a **group** obligation, really.)

**Speaker** (Combine w/ **Reflector** if there are not four group members.) Asks the facilitator questions and communicates what the team has done.

**Answer these questions.** The numbers are from the **Deduction Through the Ages** project.

**Exercise 3.1.** Although we cannot be certain of the logical import of the connective “or” as used by Diogenes Laertius in the interpretation of Chrysippus, consider two common uses of “or” in language today:

- (I) For \$7.95 the meal comes with either soup *or* salad.
- (II) As a graduation requirement in computer science, a student must pass either a physics course *or* a chemistry course.
  - (a) In statement (I) is the conclusion valid, namely that the meal costs \$7.95, if both conditions of the “or” statement are met, i.e., both soup and salad are ordered?
  - (b) In statement (II) is the conclusion valid, namely that a student fulfills a graduation requirement in science, if both conditions of the “or” statement are met, namely the student passes both a course in physics and a course in chemistry?
  - (c) Today we say that the statement “ $A$  or  $B$ ” represents the *exclusive or* if the truth of “ $A$  or  $B$ ” excludes the possibility that both  $A$  and  $B$  are true. Similarly, “ $A$  or  $B$ ” represents the *inclusive or* if the truth of “ $A$  or  $B$ ” includes the possibility that both  $A$  and  $B$  are true. In the fourth rule of inference, Chrysippus states “Either the first or the second. The first. Therefore, not the second.” Must the conclusion “Therefore not the second” necessarily follow for the “inclusive or”? the “exclusive or”? Explain your answer with a concrete example, providing actual statements for “the first” and “the second.”
  - (d) Consider the following example of Chrysippus’s fifth rule of inference: “Either it is foggy or it is cold. It is not cold. Therefore, it is foggy.” Is a particular type of “or” being used? Must the conclusion “Therefore, it is foggy” necessarily follow for the “inclusive or”? for the “exclusive or”? Justify your answer.
  - (e) When Aristotle states that a proposition is either true or false, what type of “or” is being used?

**Exercise 3.2.**

- (a) The major premise of the first rule of inference is an example of an “if-then” statement. Consider the statement: “If the roof leaks, then the house will not sell.” Using the second rule of inference, what can be concluded if the house will sell? Write this as an “if-then” statement beginning with “If the house will sell, . . . .”
- (b) Now letting  $A$  and  $B$  denote general propositions, rewrite “If  $A$ , then  $B$ ” as a different “if-then” statement that generalizes the example in part (a).

**Exercise 3.3.**

- (a) The major premise of the third rule of inference is the negation (not the case) of an “and” statement. Rewrite “It is not the case both that it is day and it is night” as an “if-then” statement so that when the pattern of the first rule of inference is followed, the minor premise becomes “It is day,” and the conclusion matches that in the example for the third rule of inference.
- (b) Now, for elementary propositions  $A$  and  $B$ , rewrite “It is not the case that both  $A$  and  $B$ ” as an “if-then” statement following the example in part (a).

**Exercise 3.4.** The major premise of the fourth rule of inference is an “exclusive or” statement. Suppose that “either  $A$  or  $B$ , but not both” is given and that  $A$  is true. What can be concluded about  $B$ ? Justify your answer.

**Exercise 3.5.** For the inclusive or, write “either  $A$  or  $B$ ” as an “if-then” statement following the pattern of the fifth rule of inference.

**Exercise 3.6.** Speculate on whether every “if-then” statement can be written as an “and” statement. Speculate on whether every “if-then” statement can be written as an “exclusive or” statement, or as an “inclusive or” statement. Apply your conjectures to the statement “if the roof leaks, then the house will not sell.” At the end of the project in Exercise 8.8 we will return to this question.