gCounting

2023-11-28

Introduction

This group assignment includes work on proof by induction and proof by strong induction.

Assignment Goals

Learning Outcomes After completing this group assignment, each student is expected to be able to

- Expand some recurrence relations
- Do some induction proofs

Procedure

Assign Roles. Students should take roles they have not held recently (or, perhaps, ever):

Manager Move discussion forward.
Recorder Writes the report that will be turned in.
Reflector Monitor that everyone gets heard and is caught up. (This is a **group** obligation, really.)
Speaker (Combine w/ **Reflector** if there are not four group members.) Asks the facilitator questions and communicates what the team has done.

Answer these questions.

1. Given a class for linked-list nodes as followws

   ```
   class Node {
     public int data;
     public Node next;
   }
   ```

   Write a *recursive* function that takes a `Node` as its parameter and returns the sum of the values in the list.

2. Consider the *recurrence relation*
   $u_k = u_{k-1} + 2k$
   $u_0 = 0$

   (a) Write out u_i for $i \in \{1, \ldots, 6\}$

   (b) Write out a *telescoping sum* by subtracting the lower-order term from the right-hand side for the six terms you calculated above.

   $u_k - u_{k-1} = 2 \times (k)$
   $u_{k-1} - u_{k-2} = 2 \times (k-1)$
   $u_{k-2} - u_{k-3} = 2 \times (k-2)$
   ...

(c) Using the cancelling telescope, find a sum for u_k in terms of u_0.

(d) Prove that $u_k = k(k+1)$.

3. Consider a pile of 4¢ and 11¢ stamps.

   (a) What values of postage can be made, exactly, with just these stamps?

   (b) Prove, using *strong* induction, that your answer above is correct.

   (c) Prove using *weak* induction, that your answer to (a) above is correct. Pay close attention to the **inductive hypotheses** and how they are different in both proofs.

4. There is a bin with 10 each of lemon, cinnamon, and rose flavored chocolates. How many chocolates must be drawn to assure that:

   (a) At least two different flavors are picked?

   (b) At least three of *one* flavor are picked?

   (c) No flavor is missing from the chocolates picked?

5. Use strong induction to show that every positive integer n can be written as a sum of distinct powers of two, that is, as a sum of a subset of the integers $2^0 = 1, 2^2 = 2, 2^2 = 4$, and so on. [Hint: For the inductive step, separately consider the case where $k+1$ is even and where it is odd. When it is even, note that $\frac{k+1}{2}$ is an integer.]