Introduction

This group assignment wraps up the **Deduction Through the Ages** project. You will work, in your group, on some of the questions in the readings and look at using modern notation with *truth tables* to **prove** some logical equivalences.

Note

- By **design**, this assignment includes questions about topics that you have not yet seen in lecture. This encourages *creative* exploration and *constructing* your own mental models.
- Your group is expected to work together on *each* question. I **must** hear the group discussing the questions and the answers. Remaining silent, even if you are following along, is choosing *not* to participate.
  The goal is **not** to fill in the "right" answers as quickly as possible but rather to take the time to read, think, and discuss the material **together**. That requires communication and I must *witness* the communication.
- Constructing your own model is **short-circuited** by reading or parroting another thinker's answer. This means you should, as much as possible, use *recall* of class material, looking up definitions in *notes* or the *textbook* when necessary, and *avoid* just typing the question into a search engine on the Web. If you want more information, feel free to search **after** the group work is finished.
- Each student in the group has more or less experience with the topics in the assignment. If you have greater familiarity with the topics, *please* hold back a little bit so others can engage with building their own knowledge. You, too, must engage because there is always more to learn.

Assignment Goals

Learning Outcomes After completing this group assignment, each student is expected to be able to

- Be able to identify Wittgetstein's *truth grounds* for logical expressions.
- Prove **deMorgan's Law** with a truth table.
- Rewrite an arbitrary *implication* as an equivalent *disjunction* and an equivalent *conjunction*.

Procedure

Get out paper. The group will turn in *one* document. Make sure all participating members' names are on the page.
Copy each question before the answer. This documents the answer and makes the page a stand-alone study guide.
Assign Roles. Students should take roles they have not held recently (or, perhaps, ever):

> **Manager** Move discussion forward.
> **Recorder** Writes the report that will be turned in.
> **Reflector** Monitor that everyone gets heard and is caught up. (This is a **group** obligation, really.)
> **Speaker** (Combine w/ **Reflector** if there are not four group members.) Asks the facilitator questions and communicates what the team has done.

Answer these questions. The numbers are from the **Deduction Through the Ages** project.

1. Dr Ladd is a stickler about the formatting of a *proof*. If you were asked to prove that $(a \wedge b) \vee (c \wedge b)$ is logically equivalent to $(a \vee c) \wedge b$, what would the **first** and the **last** lines of the proof be?

2. Comparing the equivalence in the previous question to rules for rewriting regular *algebraic* expressions and assuming the equivalence is true, what name might you give to the rule?

3. Wittgenstein introduced the term *truth-grounds* for a proposition to indicate the "truth-possibilities of its truth-arguments that make it true". (Wittgenstein's writing is nothing if not difficult to decipher.) What does *truth-grounds* mean?

4. Find the truth-grounds for each of the following logic expressions, written in *modern* notation.
   (a) $p \vee q$
   (b) $p \wedge q$
   (c) $\neg r \vee \neg s$
   (d) $t \Rightarrow u$
   (e) $\neg t \vee u$
   (f) $v \vee \neg w$
   (g) $\neg(t \wedge \neg u)$
   (h) $\neg(r \wedge s)$
5. Make note of which of the expressions in the above have the same truth-grounds.
6. Write a *truth table* for each of the following expressions. Which compound propositions are *logically equivalent* ($\equiv$)? Compare to problem 4 above.
   (a) $r \wedge s$
   (b) $\neg(r \wedge s)$
   (c) $\neg r \vee \neg s$
7. Can an *inclusive* **or** statement always be rewritten as a logically equivalent **if-then** statement? Justify your answer with a reference to appropriate truth tables.
8. **Prove** one of *deMorgan's Laws*: the negation of a *disjunction* is equivalent to the *conjunction* of the negations of the terms. Write a full proof and use a truth table.