# gStrongInduction — Induction and Strong Induction

2023-11-07

## Introduction

This group assignment includes work on proof by induction and proof by strong induction.

## Assignment Goals

**Learning Outcomes** After completing this group assignment, each student is expected to be able to

- Do some *strong* induction proofs.
- Do some induction proofs

## Procedure

**Assign Roles.** Students should take roles they have not held recently (or, perhaps, ever):

> **Manager** Move discussion forward.
> **Recorder** Writes the report that will be turned in.
> **Reflector** Monitor that everyone gets heard and is caught up. (This is a **group** obligation, really.)
> **Speaker** (Combine w/ **Reflector** if there are not four group members.) Asks the facilitator questions and communicates what the team has done.

**Answer these questions.**

1. Consider the *recurrence relation*
   $u_k = u_{k-1} + (2k+1)$
   $u_0 = 1$
   (a) Write out $u_i$ for $i \in \{1, \ldots, 6\}$
   (b) Write out a *telescoping sum* by subtracting the lower-order term from the right-hand side for the six terms you calculated above.

   $u_1 = u_0 + 2(1) + 1$
   $u_1 - u_0 = 2(1) + 1$
   (c) Using the cancelling telescope, find a sum for $u_n$ in terms of $u_0$.
   (d) Prove that $u_n = n^2$.
2. **Explain**, in English, what it means to say that some piece of code, with input size $j$, is $O(j^2)$? Include what happens if the input size is *doubled* or *tripled*.
3. **Prove** that $\forall k \in \mathbb{Z}^{\geq 0}\ square(k) \Rightarrow \neg square(k+2)$.

4. Cantor's Diagonalization for Computer Scientists:

   $|\{\text{Deciders}\}| < |\mathbb{P}(\{0,1\}^*)$

   Please explain, in English, what it means if the stated inequality holds? What has been *proved* when the whole diagonalization proof is done?

5. While proving the above inequality, a bijection, $f : \mathbb{Z}^+ \to \mathbb{P}(\{0,1\}^*)$, is represented by a table:
   (a) **Explain** why we assume such a bijection exists.
   (b) What label is give to each *row* in the table? How are the labels *ordered*?
   (c) What label is give to each *column* in the table? How are the labels *ordered*?
   (d) What is the *type* of each entry in the table? What do the possible *values* of any given **entry** in the table **mean**?
   (e) For any $j, k \in \mathbb{Z}^+$, what does it mean if $f(j)_k = 0$?
   (f) For any $j, k \in \mathbb{Z}^+$, what does it mean if $f(j)_k = 1$?
   (g) What is the *type* of $f(j)$? What does it mean?
   (h) What is the *type* of $\overline{f(j)}$? What would it mean?