# Introduction

Students will work in groups using propositional logic, truth tables, and logic operations.

# Learning Outcomes

When completing this assignment, students should be able to

- **Use** named rules of inference.
- **Set-up** different types of proof.
- **Prove** theorems with different methods of proof.

**Get out paper.**

**Copy each question before the answer.**

n**Assign Roles.** Students should take roles they have not held recently (or, perhaps, ever).

**Answer these questions.**

*A proof of a proposition is a convincing argument that the proposition is true.*

1. Given $P_1 \land P_2 \land P_3 \Rightarrow Q$ What is the contrapositive?
2. What is a *binary language*?
3. **State** the Halting Problem: What is the *decider* in question, what is the *input* to the decider, and *what* do you know about the decider in question?
4. Given the set $C = \{\spadesuit, \heartsuit, \clubsuit, \diamondsuit\}$, describe how you could **prove** $|C| = 4$.
5. How could you represent the *subsets* of $C$ as bit strings? How many bits would you need per subset? How would you assign meaning to the *order* of the bits because $C$ is unordered?
6. **Prove** $|\mathbb{P}(C)| = |\mathbb{Z}_{16}|$ using a *T-table* to express the function.
7. **Explain**, in English, what it means to say that some piece of code, with input size $j$, is $O(j^2)$? Include what happens if the input size is *doubled* or *tripled*.
8. Consider proving the following summation is equivalent to the given *closed-form* for all $g \geq 0$:
   $\sum_{i=0}^{g} i^3 = \frac{g^4+2g^3+g^2}{4}$
   (a) Before working on proving the above, prove that the closed-form is always an integer by **proving** that $\forall g \in \mathbb{Z}^{\geq 0} \quad 4 \mid g^4 + 2g^3 + g^2$
   **Note:** this can be done by induction or by algebra, your choice.
   (b) **Define** a *predicate*, $R(g)$, such that the proof of the above equivalence is the same as the proof of $\forall g \in \mathbb{Z}^{\geq 0}\ R(g)$.
   (c) **Prove** that $R(k)$ is true for all $k \in \{0, 1, 2\}$.
   (d) State and **prove** the *basis* of proving $R(g)$ for all non-negative integers.
   (e) **State** the *inductive hypothesis* without using your predicate.
   (f) State and **prove** the *inductive hypothesis* for the original equivalence; you may use your predicate if you wish. Stop after the *inductive hypothesis* is proved.
   (g) Write the final *therefore* for proving that the original equivalence holds for all non-negative numbers.
   (h) Explain, in English, your own words, what the final *therefore* statement actually means.
9. Dr. Ladd has written a piece of code that, for an input of size $g$, executes $\sum_{i=0}^{g} i^3$ instructions. The running time of the code is $O(f(g))$ for some function $f$. What is the smallest $f(g)$ you can give?