Introduction

Students will work in groups using propositional logic, truth tables, and logic operations.

Assignment Goals

- Prerequisite Knowledge
  - Know what a *proposition* and a *predicate* are.
  - Knowledge of *truth tables*
- Learning Outcomes
  - Show logical equivalence by showing two columns of a truth table are the same.
  - Prove deMorgan's Law and distributive rule for logic operations.

Procedure

Get out paper. The group will turn in *one* document. Place all the names of group members attending atop the page.
Copy each question before the answer. This makes the answer sheet a worthwhile study guide w/o the worksheet. The answers written down should be *discussed* by the group and represent the *consensus* of the group. Sometimes a *picture*, a *graph*, or something similar is the most appropriate way to answer a question.
Assign Roles. Students should take roles they have not held recently (or, perhaps, ever):

Manager Move discussion forward.
Reflector Monitor that everyone gets heard and is caught up. (This is a **group** obligation, really.)
Recorder Writes the report.

Answer these questions.
A proof of a proposition is a convincing argument that the proposition is true.

1. **Translate** the following into English: $\forall n \in \mathbb{Z}^+ \; \forall k \in \{\omega : \omega \mid n\} \; k \nmid n + 1$
2. For the numbers $h \in \mathbb{Z}_{13}$, calculate h^2 **mod** 4. What pattern do you see in the results?
   Possibly useful notation: $\equiv_n$ means that the expressions on either side are "equivalent modulo n" $69 \equiv_4 13$ because 69 **mod** $4 = 1$ and 13 **mod** $4 = 1$. Note that $a \mid b$ is the same as $b \equiv_a 0$.
3. **How** would you prove that 7 is prime? Define the predicate $prime$ and describe your approach in English. *Then* prove that 7 is prime.
4. **Prove** that the sum of two *rational* numbers is, itself, rational. Begin by rewritting the theorem in mathematical logic terms and use the definition of $\mathbb{Q}$.
5. **Prove** that $\forall b \in \mathbb{Z} \; odd(b) \Leftrightarrow odd(b^2)$. Note: How many proofs does this entail?
6. One of the following statements is true, the other false. Decide which is which and prove the true one and disprove the false one:
   (a) If $rational(x - y)$ and $rational(x)$ then $rational(y)$.
   (b) If $rational(xy)$ and $rational(x)$ then $rational(y)$.