gReview

2023-12-05

Introduction

This group assignment includes review across the semester

Assignment Goals

Learning Outcomes After completing this group assignment, each student is expected to be able to

- Provide definitions for terms used in class.
- Do some proofs.

Procedure

Assign Roles. Students should take roles they have not held recently (or, perhaps, ever):

Manager Move discussion forward.
Recorder Writes the report that will be turned in.
Reflector Monitor that everyone gets heard and is caught up. (This is a **group** obligation, really.)
Speaker (Combine w/ **Reflector** if there are not four group members.) Asks the facilitator questions and communicates what the team has done.

Answer these questions.

1. Give a precise definition of
   (a) the *powerset* of the set S, $\mathbb{P}(S)$
   (b) the *countable* predicate
   (c) $s \mid t$
   (d) *predicate*
   (e) *proposition*
2. There is a bin with 10 each of lemon, cinnamon, and rose flavored chocolates. How many chocolates must be drawn to assure that:
   (a) At least two different flavors are picked?
   (b) At least three of *one* flavor are picked?
   (c) No flavor is missing from the chocolates picked?
3. List $\mathbb{P}(\{\heartsuit, \star, \circledcirc\})$.
4. How can you prove that two sets have the same cardinality?

5. Use a *truth table* to prove that the *inverse* and *converse* of $p \Rightarrow q$ are logically equivalent. For full credit make sure to format it as we write proofs.

6. What is the *contrapositive* of $\forall n \in \mathbb{Z}^{\geq 0} \forall p \in \{\text{prime}\} \ \ p \mid n^2 \Rightarrow p \mid n$

7. **Prove** that $\sum_{i=1}^{n} (2i - 1)$ is n^2 for all positive integers, n.

8. Use strong induction to show that every positive integer n can be written as a sum of distinct powers of two, that is, as a sum of a subset of the integers $2^0 = 1, 2^2 = 2, 2^2 = 4$, and so on. [Hint: For the inductive step, separately consider the case where $k + 1$ is even and where it is odd. When it is even, note that $\frac{k+1}{2}$ is an integer.]

9. **Prove** that for any sets A and B, $countable(A) \wedge countable(B) \Rightarrow countable(A \cup B)$.

10. Let $odd = \{z \in \mathbb{Z} | z \text{ is odd.}\}$. **Prove** that $|odd| = |\mathbb{Z}|$.

11. We have proved that for sets A and B, $A \subset B$ implies $|A| \leq |B|$. Explain **why** the relationship between the cardinalities must be $\leq$ while the relationship between the two sets is $\subset$.

12. Write a short paragraph arguing that the number of *deciders* is **countable**. Include what a decider *is*, how we represent them in this class, and then give your argument.

13. Using our language shorthand, represent the set of *all* **decimal** strings.

14. Give a precise definition of
    (a) $\mathbb{Q}$
    (b) *alphabet*, *string*, *language*
    (c) *countable*
    (d) the *even* predicate