gStrongInduction — Induction and Strong Induction

2024-04-09

Introduction

This group assignment includes work on proof by induction and proof by strong induction.

Assignment Goals

Learning Outcomes After completing this group assignment, each student is expected to be able to

- Do some *strong* induction proofs.
- Do some induction proofs

Procedure

Assign Roles. Students should take roles they have not held recently (or, perhaps, ever):

> **Manager** Move discussion forward.
> **Recorder** Writes the report that will be turned in.
> **Reflector** Monitor that everyone gets heard and is caught up. (This is a **group** obligation, really.)
> **Speaker** (Combine w/ **Reflector** if there are not four group members.) Asks the facilitator questions and communicates what the team has done.

Answer these questions.

1. What does it mean to say w is a *prime* number? Use appropriate quantifiers in your definition.
2. Define a Java *predicate* `isPrime(int u)` that determines whether or not `u` is prime.
3. Consider the conjecture (unproved, so far) that for all non-negative v, $4^v - 1$ is a multiple of 3.
   (a) Rewrite the statement TBP as a *quantified* logic statement.
   (b) What is the basis value for this statement?
   (c) Define a **predicate**, $T(v)$, to represent the thing you want to prove being *true* for the integer v.
   (d) Write the **basis** for an inductive proof of this conjecture using your T function.
   (e) Rewrite the *basis* without using T: expand it with its definition.
   (f) Write the **inductive hypothesis** for an inductive proof of this conjecture using your T function.
   (g) Rewrite the *inductive hypothesis* without using T: expand it with its definition.
   (h) Prove that the conjecture is true.

4. The **Fibonacci sequence** is defined as $F_0 = 0$, $F_1 = 1$, and $F_n = F_{n-1} + F_{n-2}$ after that.
   (a) List the Fibonacci numbers up to F_6.
   (b) Write a *recursive* Java function, `fib` that takes an integer, n, as a parameter and returns F_n.
   (c) Prove that $\sum_{k=0}^{n} = F_{n+2} - 1$ for all n greater than zero.
5. Consider the sequence defined as $d_i = i$ for $1 \leq i \leq 3$ and $d_{n+3} = d_{n+2} + d_{n+1} + d_n$ after that.
   [Strong Induction, Brilliant.org. Retrieved from https://brilliant.org/wiki/strong-induction/]
   (a) List d_j for j from 1 to 8.
   (b) [Strong Induction] Prove that for all $n \in \mathbb{Z}^{>0}$, $d_n < 2^n$.