1. **Prove** or **disprove**:

$$\forall p \in \{prime\}, s,t \in \mathbb{Z}^+ \; (p \mid st) \Rightarrow (p \mid s) \vee (p \mid t)$$

   (**Hint:** Not inductive. Rewrite the implication to something logically equivalent.)

2. Some questions about gcd and divisibility. For $x, y \in \mathbb{Z}^+$:
   (a) If $2 \mid x$ and $2 \mid y$ what, if anything, do you know about $gcd(x, y)$? Why?
   (b) If $51 \mid x$ and $51 \mid y$ what, if anything, do you know about $gcd(x, y)$? Why?
   (c) If $2 \mid x$ and $17 \mid y$ what, if anything, do you know about $gcd(x, y)$? Why?
   (d) If $2 \mid x$, $17 \mid x$, $2 \mid y$, and $17 \mid y$ what, if anything, do you know about $gcd(x, y)$? Why?

3. Prove that for all non-negative integers k, $6 \mid k^3 + 3k^2 + 2k$.

4. For the Fibonacci sequence: the sum of the first a *odd* elements is F_{2a}.

$$\forall a \in \mathbb{Z}^+ \sum_{j=1}^{a} F_{2j-1} = F_{2a}$$

   (**Hint:** Check the sums for some small values of a and look for *patterns*.)

5. Consider the $\sum_{i=0}^{n} 2 \cdot 3^i$.
   (a) Write a **recursive** Java method, `int someSum(int n)` that calculates the sum. Do **not** use the answer to the following part of the question to calculate the answer.
   (b) Prove $\forall n \in \mathbb{Z}^{\geq 0} \sum_{i=0}^{n} 2 \cdot 3^i = 3^{n+1} - 1$.

Points earned: ______
out of a possible 0 points