Learning Outcomes

Upon completing this homework, students should be able to

- **Set-up** direct, contradiction, contrapositive, and both kinds of inductive proofs.
- **Write** a two-colunm direct and contradiction proofs.
- **Prove** theorems given type of proof to use.
- **Prove** theorems after *choosing* proof type.

Assignment

A proof of a proposition is a convincing argument that the proposition is true.

1. **Translate** the following into English: $\forall n \in \mathbb{Z}^{+}\ \forall k \in \{\omega : \omega \mid n\}\ k \not| n+1$
2. Set up a *direct* proof that 19 is *prime*.
   (a) State the *definition* of *prime(k)*.
   (b) Give the first line of the direct proof.
   (c) Give the last line of this direct proof.
3. Set up a proof by *contradiction* that $\sqrt{19}$ is irrational:
   (a) State the *definition* of *irrational(k)*.
   (b) Give the first line of the proof.
   (c) State the assumption made *for sake of contradiction*.
   (d) Give the last line of this proof.
4. If you are asked to prove that "The sum of any two *rational* numbers is *rational*"
   (a) What type of proof will you use? Justify your answer in a sentence.
   (b) Translate the statement into logic and state the first line of the proof.
5. If you are asked to prove that "The sum of any *rational* number and an *irrational* number is *irrational*"
   (a) What type of proof will you use? Justify your answer in a sentence.
   (b) Translate the statement into logic and state the first line of the proof.
6. If you are asked to prove that "For all non-negative integers, *h*, $19|h^{19} - h$"
   (a) What type of proof will you use? Justify your answer in a sentence.
   (b) Translate the statement into logic and state the first line of the proof.
7. **Prove**, using *weak induction*, $\forall b \in \mathbb{Z}^{+} 3 \mid b^3 + 6b^2 + 5b - 12$.
8. **Prove** that $\forall n \in \mathbb{Z}^{\geq 0} odd(n^2) \Rightarrow odd(n)$.
9. **Prove** by *contradiction* that $\sqrt{19}$ is irrational. You may assume $\forall z\ 19 \mid z^2 \Rightarrow 19 \mid z$.
10. **Prove** that the following closed-form for the summation holds for all $n \in \mathbb{Z}^{+}$:
    $\sum_{i=1}^{n} i^2 - 1 = \frac{n(n+1)(2n+1)-6n}{6}$

Submit your answers electronically, in a commonly readable format (*e.g.* `.pdf`, `.txt`, `.docx`), through BrightSpace.