This lab has **six (6)** checkpoints.

Learning Outcomes

Upon completing this lab, students should be able to

- Work with *sets*, *set operators*, *set relations*, and *power sets*.

Introduction

Use these definitions while answering the following questions.

$$
\begin{array}{lcl}
A & = & \{x | x \in \text{the English alphabet}\} \\
2\mathbb{Z} & = & \{x \in \mathbb{Z} \ni 2 \mid x\} \\
\mathbb{Z}_7 & = & \{0, 1, 2, 3, 4, 5, 6\} \\
\mathbb{Z}_5 & = & \{0, 1, 2, 3, 4\} \\
V & = & \{a, e, i, o, u\} \\
11\mathbb{Z} & = & \{y \in \mathbb{Z} \ni 11 \mid y\} \\
R & = & \{red, orange, yellow, green, blue, indigo, violet\} \\
CYMK & = & \{cyan, yellow, magenta, black\} \\
C & = & \{\clubsuit, \heartsuit, \spadesuit\}
\end{array}
$$

1. Answer the following:
   (a) Which sets above are *infinite*?
   (b) $A - V = ?$
   (c) $\mathbb{Z}_7 \cap \{z \in \mathbb{Z}^+ \ni even(z)\}$ = ?
   (d) $CYMK \cup R$ = ?
   (e) True or false: $\emptyset \in \mathbb{P}(R)$?
   (f) True or false: $R \subseteq \mathbb{P}(R)$?
   (g) $V - A = ?$
   (h) $\mathbb{P}(C) = ?$
   (i) $|\mathbb{P}(A)| = ?$
2. Use the *set builder* notation to describe each of the following
   (a) $T = \{\text{all multiples of three}\}$
   (b) $F = \{\text{all multiples of five}\}$
   (c) $T \cap F$
3. Given two *non-empty*, *disjoint* sets, Y and B,
   (a) What is the cardinality of $Y \cap B$?
   (b) What is the cardinality of $Y \cup B$?
4. If there exists two *finite* sets, X and M, such that $M = \mathbb{P}(X)$, what do you know about $|M|$, $|X|$, and the relationship between them?
5. Remainders:

(a) Write one line in Java using two `int` variables, `remainder` and `value`. Your line should assign the *remainer* left when `value` is divided by `7` to the variable `remainder`.

(b) What is the *set* of possible values that `remainder` might be set to by your line?

6. Consider A.

   (a) Explain how you know how many subsets of A have zero elements.

   (b) Explain how you would determine how many subsets of A have exactly 25 elements.

   (c) What is the *compliment* of $V = \{\text{English vowels}\}$ if A is the universe?

7. Consider an arbitrary set G and its relationship to $\mathbb{P}(G)$:

   (a) When (if ever) is $G \in \mathbb{P}(G)$? Explain your answer, in particular explaining how you know you have *all* of the cases.

   (b) When (if ever) is $G \subseteq \mathbb{P}(G)$? Explain your answer, in particular explaining how you know you have *all* of the cases.