The following test is *closed book*; you may neither give nor receive assistance during this exam. Do not discuss the content of this exam until it is turned back in class.
Read each question *carefully* and answer it in the space provided. If you require more room, continue on the back of the page after clearly labeling what question the answer goes with.

Name: ______________________

Set-builder notation: The definition of *union*, in set-builder notation would be: $A \cup B = \{x | x \in A \vee x \in B\}$; it rewrites set operations into logic statements.

1. Describe, in simple English, what the set $\mathbb{Z} \times \mathbb{Z}$ is. (5)

2. Is the set $\mathbb{Z}^{>7} \times \mathbb{Z}^{>7}$ countable? Justify your answer. (5)

3. Give the *set-builder notation* defining $A \subset B$. (5)

4. Using the *set builder notation*, prove DeMorgan's Law for taking the compliment of the union of two sets. (10)

5. Consider $3\mathbb{Z} = \{z | z \text{ is a multiple of } 3.\}$. *Prove* that $|3\mathbb{Z}| = |\mathbb{Z}|$. (5)

6. Give an example of two sets, A and B, such that $|A| = |B|$ and A is a *proper* subset of B. (5)

7. Give an example of two sets, A and B, such that $|A| < |B|$ and A is a *proper* subset of B. (5)

8. Give a function from $\mathbb{Z}^+$ to $odd(\mathbb{Z}^+)$ that has **exactly** the given property *or* explain why no such function is possible.
   (a) Only *injective* (3)
   (b) Only *surjective* (3)
   (c) *bijective* (4)

Points earned: ________
out of a possible 50 points

9. Consider the *predicate* $prime$ for integers.
   (a) What is the *domain* of the function? (3)
   (b) What is the *codomain* of the function? (3)
   (c) What is the *signature* of $prime$? (3)
   (d) What is the *range* of the function? Justify your answer. (3)
   (e) Is this function *injective*, *surjective*, *bijective*, or none of these? Justify your answer. (3)

10. Define an *algorithm* that returns `true` if the function, `f`, is *onto* from finite set `A` to finite set `B` and `false` otherwise. This is an *algorithm* so you can define your own notation for things like the cardinality of a set if you need to; make sure your notation's meaning is clear. (10)

```
boolean isOnto(set A = a_1, a_2, ..., a_m,
               set B = b_1, b_2, ..., b_n,
               B f(A a)) {

}
```

11. Prove that the union of any two *countable* sets is *countable*. (12)

12. Consider the set of ordered pairs of positive integers, $\mathbb{Z}^+ \times \mathbb{Z}^+$. Prove that $\mathbb{Z}^+ \times \mathbb{Z}^+$ is *countable*. (13)

Points earned: ______
out of a possible 50 points