1. You have see a great number of *symbols* in lecture already. Two of them are $\in$ and $\ni$. Which of these symbols is to be read "such that"?

Solution:

$\in$ - reads as "in" or "is a member of" with a set: $7 \in \mathbb{Z}$
$\ni$ - reads as "such that": $even(x)$ means there exists $y \in \mathbb{Z} \ni 2y = x$

2. **Expand** each of the following by writing out *all* of the terms with appropriate operators between them.

(a) $\sum_{i=1}^{4} 10 \times i$

Solution:

$10 \times 1 + 10 \times 2 + 10 \times 3 + 10 \times 4$
$10 + 20 + 30 + 40 = 100$

(b) $\sum_{j=3}^{5} j^2$

Solution:

$3^2 + 4^2 + 5^2$
$9 + 16 + 25 = 50$

(c) $\sum_{k=100}^{104} k \mod 3$

Solution:

$(100 \mod 3) + (101 \mod 3) +$
$(102 \mod 3) + (103 \mod 3) +$
$(104 \mod 3)$
$1 + 2 + 0 + 1 + 2 = 6$

(d) $\sum_{p=4}^{4} 2^p$

Solution:

2^4
16

(e) $\sum_{q=0}^{3} 2^q$

Solution:

$2^0 + 2^1 + 2^2 + 2^3$
$1 + 2 + 4 + 8 = 15$

(f) $\prod_{r=1}^{4} r + 1$

Solution:

$(1+1) \times (2+1) \times (3+1) \times (4+1)$
$2 \times 3 \times 4 \times 5 = 120$

(g) $\prod_{s=1}^{6} 10^s$

Solution:

$10^1 \times 10^2 \times 10^3 \times 10^4 \times 10^5 \times 10^6$
$10^{\sum_{i=1}^{6} i} = 10^{21}$

(h) $\prod_{t=1}^{3} t \cdot 2$

Solution:

$(2 \cdot 1) \times (2 \cdot 2) \times (2 \cdot 3)$
$2 \times 4 \times 6 = 48$

3. Calculate the result for each of the sums and products in the previous question.

 Points earned: ______
out of a possible 0 points

Solution:
See above.

4. What is the *difference* between 1.(e) and 1.(d)?

Solution:
$2^4 - (2^0 + 2^1 + 2^2 + 2^3) = 1$

5. Given what you know about the symbol $\in$, calculate the following value: $1 + \prod_{p \in \{2,3,5,7\}} p$

Solution:
$1 + (2 \times 3 \times 5 \times 7) = 211$

6. Why does the product in the previous question not *need* an "upper" bound?

Solution:
The "sequence" of multiplicands comes from the listed elements of the set. It is not counting but rather taking on each value between the curly braces.

7. Is the answer to 5. above divisible by *any* of the primes in the set used for the product?

Solution:
No. Each leaves a remainder of 1.

8. Evaluate each of the following expressions. Pay careful attention to the *type* of each of your answers.

(a) $17 \mid 51$

Solution:
Boolean: true
($\mid$ is "divides")

(b) $144 \mod 41$

Solution: Integer: 21
($3 \times 41 = 123 + 21 = 144$)

Points earned: ______
out of a possible 0 points

(c) The *greatest common divisor* of 288 and 84

Solution:

Integer: 12

(d) $v^5 \cdot v^8$

Solution:

Same as v: v^{13}

(e) $18 \nmid 88$ (What do you *think* it means?)

Solution:

Boolean: not ($18 \mid 88$) = not (false) = true

(f) $\frac{w^{71}}{w^{75}}$

Solution:

Same as w: $\frac{1}{w^4}$

9. What is the *definition* of the *divides* predicate? That is: **Define** $a \mid b$. (Either definition presented in class is acceptable.)

Solution:

$a \mid b$ ::= $b \mod a = 0$ *-or-*
$a \mid b$ ::= there exists an integer, k, such that $ak = b$

Points earned: ______
out of a possible 0 points